24.19. Model: The generalized formula of Balmer predicts a series of spectral lines in the hydrogen spectrum.
Solve: **(a)** The generalized formula of Balmer

$$\lambda = \frac{91.18 \text{ m}}{\left(\frac{1}{m^2} - \frac{1}{n^2}\right)}$$

with $m = 1$ and $n > 1$ accounts for a series of spectral lines. This series is called the Lyman series and the first two members are

$$\lambda_1 = \frac{91.18 \text{ m}}{\left(1 - \frac{1}{2^2}\right)} = 121.6 \text{ nm} \qquad \lambda_2 = \frac{91.18 \text{ nm}}{\left(1 - \frac{1}{3^2}\right)} = 102.6 \text{ nm}$$

For $n = 4$ and $n = 5$, $\lambda_3 = 97.3$ nm and $\lambda_4 = 95.0$ nm.
(b) The Lyman series converges when $n \to \infty$. This means $1/n^2 \to 0$ and $\lambda \to 91.18$ nm.
(c) For a diffraction grating, the condition for bright (constructive interference) fringes is $d \sin\theta_p = p\lambda$, where p = 1, 2, 3, … For first-order diffraction, this equation simplifies to $d \sin\theta = \lambda$. For the first and second members of the Lyman series, the above condition is $d \sin\theta_1 = \lambda_1 = 121.6$ nm and $d \sin\theta_2 = \lambda_2 = 102.6$ nm. Dividing these two equations yields

$$\sin\theta_2 = \left(\frac{102.6 \text{ nm}}{121.6 \text{ nm}}\right)\sin\theta_1 = (0.84375)\sin\theta_1$$

The distance from the center to the first maximum is $y = L\tan\theta$. Thus,

$$\tan\theta_1 = \frac{y_1}{L} = \frac{0.376 \text{ m}}{1.5 \text{ m}} \Rightarrow \theta_1 = 14.072° \Rightarrow \sin\theta_2 = (0.84375)\sin(14.072°) \Rightarrow \theta_2 = 11.84°$$

Applying the position formula once again,

$$y_2 = L\tan\theta_2 = (1.5 \text{ m})\tan(11.84°) = 0.314 \text{ m} = 31.4 \text{ cm}$$